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A brief comment on instanton-like singularities and cosmological horizons

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Abstract: We argue that in the presence of instanton-like singularities, the existence of cosmological horizons can become frame-dependent, i.e. a horizon which appears in Einstein frame may not appear in string frame. We speculate on the relation between instanton-like singularities and the formulation of quantum gravity in de Sitter space.

Keywords: Superstrings and Heterotic Strings, M-Theory.

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1. Introduction

Recently, one of the most interesting questions to be examined by string theory is the question of how one formulates quantum gravity in de Sitter space (see [1], and references therein). In quantum gravity, it is not yet clear how one can define any gauge-invariant quantities aside from the S-matrix. However, de Sitter space has cosmological horizons which seem to make it impossible to define an S-matrix. This problem seems to be made more interesting by recent astronomical observations which are consistent with a positive cosmological constant. And it has been shown ([2, 3]) that even if the cosmological constant is in fact zero, a universe with eternally accelerating expansion (as in some quintessence models) will still exhibit cosmological horizons which make the formulation of quantum gravity problematic.

It is important to first realize that this is not truly a phenomenological problem. Certainly there is a sense in which the scattering amplitudes computed to explain everything from accelerator experiments to planetary motion are reasonable approximations, despite the potential presence of cosmological horizons. In any case, current astronomical experiments are perfectly consistent with a universe whose expansion will eventually stop accelerating at some time in the future. In such a universe there are no cosmological horizons, and thus no obstruction to the defining quantum gravity in terms of an S-matrix as a formal matter of principle.

Secondly, it is important to note that the question of cosmological horizons does not arise only in string theory. It appears to be a general statement about quantum gravity that the only well-defined gauge-invariant (and hence diffeomorphism-invariant) quantities that are well-understood are S-matrices, which seem problematic in the presence of cosmological horizons.

But despite the fact that these cosmological horizons do not cause a phenomenological problem, they do raise an interesting theoretical and aesthetic question, namely, does the mere existence of quantum gravity require that the expansion of the universe must stop accelerating at some point in the future?

A very important development related to this problem is the recent study of instanton-like singularities[1] [4]-[6] (see also [7], for a related discussion). We will argue that in the presence of such singularities, the transition between Einstein frame and string frame becomes much more subtle. In particular we will claim that it is possible for an Einstein metric with cosmological horizons to map to a string frame metric with no cosmological horizon.

Note that we do not claim to have found a de Sitter space solution to string theory, nor indeed any solution with cosmological horizons. De Sitter space does not admit supersymmetry, and it is notoriously difficult to find stable solutions to string theory without supersymmetry [8, 9]. But with de Sitter solutions, the question has been whether the cosmological horizons form an obstruction in principle to finding a solution, beyond the usual complexities of breaking supersymmetry. We claim that the appearance of a cosmological horizon is not necessarily sufficient to doom any theory of quantum gravity, and that in certain circumstances it is possible to avoid any difficulties by an appropriate field redefinition.

2. Skirting the horizon

The metric of a D-dimensional de Sitter space (in conformal coordinates) may be written as

$$ds^2 = \frac{1}{\cos^2 T}(-dT^2 + d\Omega_{D-1})\,, \tag{2.1}$$

where

$$-\frac{\pi}{2} < T < \frac{\pi}{2} \tag{2.2}$$

The conformal factor $1/\cos^2 T$ is irrelevant to the causal structure of de Sitter space. One sees that the appearance of cosmological horizons is a result of the limits on the range of the conformal time T. If an observer is located at the north pole of the $(D-1)$-sphere parameterized by $d\Omega$, then a signal from the south pole will only arrive after a conformal time $\delta T = \pi$. Thus a signal from the south pole emitted at the "beginning of time" will just reach the north pole at the "end of time." A signal emitted from the south pole any later than the $T = -\pi/2$ will never reach the observer at the north pole. This is the statement that a cosmological horizon exists.

Because of the conformal factor in the metric, a particle will reach $T = \pm\pi/2$ only after an infinite proper time. This would seem to indicate that these points truly are the beginning and end of time. But the question of whether strings respect these limits depends on the metric which the string actually sees. It is well-known that the conformal factor in the metric changes when one goes from Einstein frame to string frame. We will find that for a dilaton which is singular and diverges in an appropriate way, the cosmological singularity which appears in Einstein frame becomes harmless in string frame.

[1] We are grateful to M. Costa for pointing out an unfortunate error in notation in an earlier version of this work.

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We will treat the de Sitter solution above as a solution in Einstein frame.[2] To illustrate with a simple example, consider this choice of dilaton:

$$g = g_0 \cos^4 T \,. \tag{2.3}$$

Using the standard transformation between Einstein and string frame appropriate to type-IIA supergravity, one would get a string frame solution of the form

$$\begin{aligned} & ds^2 = -dT^2 + d\Omega_{D-1} \\ & -\frac{\pi}{2} < T < \frac{\pi}{2} \,. \end{aligned} \tag{2.4}$$

The important thing to note is that in string frame, one will reach the end of time at $T = \pi/2$ in *finite* proper time. There is no curvature singularity at this point; from the D-dimensional point of view everything is perfectly smooth. As a result, there is no obstruction to simply extending the metric so that the conformal time T runs over all real values. But the fact that the range of T was limited was precisely what caused the appearance of the cosmological horizons in the first place. In the new covering space, there are no cosmological horizons. The removal of the cosmological horizon by adopting a covering space depended on the fact that one reached the end of time in a finite proper time in string frame (the precise form of the dilaton was not important, so long as the proper time became finite). Strings could then probe across this boundary in a way perhaps similar to what was seen in [10].

In order to obtain a de Sitter space solution in Einstein frame, one should expect to have a stress energy tensor of the form $8\pi T_{\mu\nu} = -\Lambda g_{\mu\nu}$, with Λ being a positive cosmological constant. The contribution of the dilaton kinetic term to the stress energy tensor for the above solution will clearly not be homogeneous, so one would have to have additional contributions from other scalars or field strengths which combine to yield a homogeneous energy tensor. For less symmetric solutions with a cosmological horizon, one would expect this condition to be relaxed.

3. Instaton-like singularity

The vanishing of the string coupling at a locus of space-time is a signal of the appearance of a singularity, which is often resolved by new degrees of freedom. From the 11-dimensional point of view, it corresponds to a singularity which looks like a cone, in which the size of the dilaton direction shrinks to zero at the locus. Another example would be certain supergravity solutions where e^{ϕ} vanishes at the core. This corresponds to the existence of a brane at the core. In our case, however, the locus on which the string coupling vanishes is localized in time, suggesting that whatever degrees of freedom appear at this singularity are in some sense instantonic.

In order for a cosmological horizon to disappear under a change of frame, it is necessary (but not sufficient) for the string coupling to vanish at the "end of time." Thus, it seems

[2]In a realistic model, de Sitter space might emerge as a 4-D effective theory when 10-D flat space is compactified on some non-trivial manifold.

that this potential mechanism for avoiding the problems of cosmological horizons depends intimately on the appearance of instanton-like singularities.

In [5] examples of such instanton-like singularities were discussed. They considered certain $\mathbb{Z}_2$ orbifolds in which the time coordinate as well as space coordinates were inverted under the $\mathbb{Z}_2$ action. For simplicity, consider an example where only one other spatial coordinate x is inverted. The t and x coordinates together form a cone, which is singular at the tip. By uplifiting to M-theory and taking the angular direction on the cone to parameterize the dilaton direction, one finds that the string coupling constant vanishes at the instanton-like singularity, exactly as is desired in the scenarios discussed here. Similarly, M-branes wrapped on the singularity yield degrees of freedom fixed to the singularity.

3.1 Frame dependence?

It seems very odd indeed that something as significant as a cosmological horizon should be frame dependent (issues relevant to the comparison of physics in different frames were discussed in [11]). Usually, physics in Einstein frame and in string frame are equivalent, but it appears that this connection may be much more subtle in the presence of instanton-like singularities.

There is another context in which one sees a somewhat similar distinction between Einstein and string frame. Consider a linear dilaton solution to bosonic (or super)string theory:

$$\begin{aligned} g^s_{\mu\nu} &= \eta_{\mu\nu} \\ e^{\phi} &= e^{ax} , \end{aligned} \tag{3.1}$$

where $a = \sqrt{(26-D)/(6\alpha')}$ and the worldsheet theory has conformal invariance. If one changes to Einstein frame, the new metric is of the form

$$g^e_{\mu\nu} = e^{-cx}\eta_{\mu\nu} , \tag{3.2}$$

where $c = 4a/(D-2)$.

In string frame, the proper distance between a point in the interior and the strong coupling region at $x = \infty$ is infinite, while in the Einstein frame that distance is finite. This well-understood case is actually quite similar to the scenarios discussed here, except that we have looked at cases where the dilaton was time-dependent instead of spatially-dependent. As a result, we have found that the proper time between a point in the interior and the weak coupling region is infinite in the Einstein frame, but finite in string frame.

3.2 New instantaneous degrees of freedom

The appearance of these new degrees of freedom behind the cosmological horizon is reminiscent of a similar phenomenon which occurs when black hole event horizons appear. Indeed, the questions about how one should formulate quantum gravity in the presence of a cosmological horizon could just as well have been asked in the presence of a black hole event horizon. If one forms a charged black hole which never completely decays through

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Hawking radiation, then it might appear that this process cannot be described by an S-matrix, since some of the infalling material falls behind a black hole horizon and never emerges to reach asymptotic infinity. But it is believed that the solution to this problem involves a microscopic description of the states of a black hole. In particular, it is believed that this microscopic description allows the states of the black hole to be included as part of the outgoing states, and thus allows the process of matter falling into a black hole to be described by an S-matrix. For certain extremal (or near-extremal) black holes, this idea can be made concrete by describing the degrees of freedom of the black hole in terms of D-branes or strings.

It might seem that the degrees of freedom on this instanton-like object which appears in this scenario might similarly provide additional outgoing states, which allow string theory in the presence of a naive cosmological horizon to still be described in terms of an S-matrix. But one significant difference between this case and the black hole case is that the strings and branes which described the microscopic black hole states were extended in time. Thus, one could easily see how they could contribute to the set of outgoing states at future infinity. But these new instanton-like degrees of freedom are fixed at a point in time at the edge of each de Sitter patch. It is not obvious that they can be "seen" in the asymptotic future. It was suggested in [5] that these states may have to be traced over when computing scattering amplitudes.

4. Conclusions

It is not clear what to make of this apparent distinction between the physics of the Einstein frame and the string frame in the presence of these instanton-like singularities. This could possibly be an indication that particle physics is a bad approximation in the presence of these instanton-like singularities and cosmological horizons. Instead, space-time is inherently "stringy." The idea that the particle approximation gives way to a string formulation is not new; it is known that Einstein gravity is unrenormalizable as a field theory, and must give way to some more fundamental theory in the ultraviolet (with string theory being the most likely candidate at the moment). But we seem to have have evidence that the particle approximation can break down in the infra-red as well. One can probe intermediate distance scales in these backgrounds with particles by performing scattering experiments in a background which is similar at the desired scale, but which has a vanishing energy density at larger scales (so that there is no cosmological horizon). But if the region to be probed is large enough, then a cosmological horizon will appear in the interaction region and the particle approximation will break down. It may be necessary instead to treat the objects to be scattered as objects in string theory (or M-theory). In the presence of certain instanton-like singularities, it seems that this description does provide for a meaningful S-matrix. Since the string frame metric is smooth with no horizons throughout the covering space, there seems to be no obstruction to strings which stretch from one de Sitter sheet to another (although this cannot be made explicit without a precise solution). Apparently string fields which are located outside each others putative cosmological horizons can nevertheless be correlated, in a sense similar to [10].

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Unfortunately, these arguments are extremely heuristic and somewhat speculative. Clearly, this is not presented as a solution to the problems of cosmological horizons. Rather, it is a potential scenario which could be useful in formulating such a solution. It would be interesting to generate backgrounds with such instanton-like singularities and cosmological horizons, and to determine if all solutions to M-theory with cosmological horizons also have instanton-like singularities which are covered by the horizon.

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